# RATIONAL USE FOR FLUTTER CALCULATION OF VALUES FOUND BY STATIC-VIBRATION TEST

by G. de Vries

SUMMARY

The report demonstrates that in cases in which the number of natural-vibration forms introduced into flutter calculation is restricted, these forms must first be simplified by neglecting the deformations of certain structural elements in the calculation of the aerodynamic forces. The appropriate physical considerations involved are explained. A method of experimentation is indicated which facilitates the selection of the structural elements used in calculation. Further development of this procedure may make it possible to obtain also numerical data for certain structural values.

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## 1. INTRODUCTION

If we measure all natural-vibration forms together with the respective generalized masses in a static vibration test, we can then calculate the critical speed from the natural values of that matrix which is obtained from the diagonal matrix of the natural-vibration forms without aerodynamic forces by addition of the general, non-symmetric matrix of the aerodynamic forces. This procedure is utilized exclusively by O.N.E.R.A. However, since we have not as yet available the computers required for introducing all forms simultaneously into calculation, we are compelled to make a suitable selection among the forms. Such selection is facilitated by considerations of energy on the basis of results of computation with one or two degrees of freedom (1). These calculations but also quite generally any calculations with a restricted number of degrees of freedom, raise the question which values should be introduced in the calculation. It turns out that these are not always the measured natural-vibration forms.

## 2. INTRODUCTION OF NATURAL-VIBRATION FORMS INTO FLUTTER CALCULATIONS

A simple example will best show the tasks resulting when the number of the natural-vibration forms used in calculation is largely restricted, e.g., to two. In a wing, let us select the basic deflection and torsion which will have the amplitude distribution in the reference section shown in Fig. 1. A simple physical consideration will show that it would be erroneous to use the measured natural-vibration forms unchanged in the calculation. This would mean that we assume the relation between control-surface and wing amplitudes as fixed or as a kinematic linkage which obviously does not exist. The results of such a calculation scarcely have any physical meaning because they belong to a substitute system which may be very different from the wing.

We need apparently to "correct" the natural-vibration forms before using them in the calculation. The manner of effecting this correction is shown when we start from a more complete equation system, i.e. add the natural-vibration form of the control-surface, in the present case. After introducing the aerodynamic forces $\lambda_{ik}$, we obtain the determinant

$$D = \begin{vmatrix} \varphi_{11} - \mu_{11} \cdot \omega^2 + \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \varphi_{22} - \mu_{22} \cdot \omega^2 + \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \varphi_{33} - \mu_{33} \cdot \omega^2 + \lambda_{33} \end{vmatrix} \quad (1)$$

in which

$\varphi_{11}$ , $\varphi_{22}$ , $\varphi_{33}$ = generalized stiffness,

$\mu_{11}$ , $\mu_{22}$ , $\mu_{33}$ = generalized mass,

$\lambda_{ik}$ = generalized aerodynamic forces,

$\omega$ = angular velocity (real for critical speed).

It is meaningful only to equate a subdeterminant with zero instead of the determinant D when the elements of a row or a column, except for one, become infinitely small. This can be accomplished if we assume added masses for the control-surface so that the latter remains, for the wing forms, in the wing plane. With this, the determinant $D_s$ of the static-vibration forms changes. We first obtain the new determinant

$$D_s' = \begin{vmatrix} \varphi_{11}-\mu_{11}\cdot\omega^2-m_{11}\cdot\omega^2 & -m_{12}\cdot\omega^2 & -m_{13}\cdot\omega^2 \\ -m_{21}\cdot\omega^2 & \varphi_{22}-\mu_{22}\cdot\omega^2-m_{22}\cdot\omega^2 & -m_{23}\cdot\omega^2 \\ -m_{31}\cdot\omega^2 & -m_{32}\cdot\omega^2 & \varphi_{33}-\mu_{33}\cdot\omega^2-m_{33}\cdot\omega^2 \end{vmatrix} \quad (2)$$

and can calculate the new natural-vibration forms in which deflection and torsion of the wing no longer have any control-surface motion. This makes it possible to carry out calculation for the wing. The result is then valid for a system contiguous to the basic system, i.e. a wing with a dynamically compensated control-surface.

Since in general a difference exists in the magnitude of the values $\mu$ of the generalized masses of control-surface and wing forms, the $\mu$-values of the wing will change, due to the small added masses on the control-surface, only within the range of the accuracy of measurement of the basic values so that calculation of the new natural-vibration forms becomes unnecessary. It is sufficient to suppress the control-surface amplitudes.

We should therefore remember that in some cases the natural-vibration forms must be changed before introducing them into calculation. However, it is not always possible to base ourselves on the difference in magnitude of the $\mu$-values in order to simplify them. In the general case, another logical approach is a better way to arrive at useable guide lines.

## 3. OBSERVATIONS ON AERODYNAMIC FORCES TO BE CONSIDERED IN FLUTTER CALCULATIONS

There is no doubt that many calculations of flutter carried out with a wing or empennage alone have shown satisfactory conformity with the results in the wind tunnel or in flight. If we consider the natural-vibration forms of an aircraft, we will note that the amplitudes of the empennage can be paralled, for example, by equally large amplitudes of the wing. This raises again the question whether the aerodynamic forces acting on the wing should or should not be introduced in the calculations of the empennage. Here the same is again true which has already been said in regard to the ailerons. If the amplitude relation of the elements considered, i.e. wing and empennage in this case, does not change under the influence of the aerodynamic forces, then the wing must also be taken into account in calculation of the empennage. This corresponds to a case of strong coupling between wing and empennage. In a weak coupling, there is no firm relation between the amplitudes and we are safe in neglecting the amplitudes of the wing.

The concept of the natural-vibration forms of the complete aircraft is momentarily abandoned with this and the system is assumed to be subdivided into a series of partial systems such as elevator, rudder and wing with their own natural-vibration forms which become linked into the natural-vibration forms of the complete aircraft.

It is therefore the coupling which is of interest in the cases discussed and in many others. We can, as in the example of the aileron, state the static-vibration determinant for many natural-vibration forms and draw conclusions from it by introducing added masses or stiffness in a structural part and observing the change of amplitude relations after solving the determinant. However, it is simpler to gain a clear insight into this already during the static-vibration test, especially since this is possible without appreciable additional effort /expense?/ as will be shown.

## 4. EVALUATION OF COUPLING VALUES IN STATIC-VIBRATION TESTS

The following investigations are carried out on the system represented in Fig. 2 which consists of two simple vibrators with a coupling spring. The vibrators shall here correspond to the structural elements wing and empennage. Let $m_1$ and $m_2$ be the masses of the vibrators and $c_1$, $c_2$ and $c$ the spring constants. Without coupling, the vibrators have the natural frequencies $\omega_1^* = \sqrt{c_1/m_1}$ and/or $\omega_2^* = \sqrt{c_2/m_2}$. The vibration equations for the system with coupling read

$$m_1 \ddot{y}_1 + c_1 y_1 - c (y_2 - y_1) = o \quad ,$$

$$m_2 \ddot{y}_2 + c_2 y_2 + c (y_2 - y_1) = o \quad . \tag{3}$$

With the basic formulation

$$y_1 = A_1 e^{i\omega t} \quad , \quad y_2 = A_2 e^{i\omega t}$$

there results for the two natural frequencies $\omega_1$ and $\omega_2$ of the system the equation

$$\omega^4 - m_1 m_2 - \omega^2 \left\{ m_1 (c_2 + c) + m_2 (c_1 + c) \right\} + c_1 c_2 + (c_1 + c_2) = o \quad . \tag{4}$$

For each of the two amplitudes belonging to each of the two natural frequencies, the equation then reads

$$\frac{A_1(\omega_k)}{A_2(\omega_k)} = 1 + \frac{c_2 - m_2 \omega_k^2}{c} \quad , \quad k = 1 \, , \, 2 \quad . \tag{5}$$

The spring constant $c$ of the coupling spring was changed and $m_1$ and $m_2$ as well as $\omega_1$ and $\omega_2$ were kept constant. The corresponding values $c_1$ and $c_2$ were selected in correspondence with this. More detailed indications can be found in Fig. 2. We now identify the two vibrating masses with the mass of the wing and/or the mass of the empennage and assume them to be connected by a mass-less beam (fuselage). In Fig. 2, there are further entered, for each of the two natural frequencies $\omega_1$ and $\omega_2$ in each case for three values of $c$, the amplitudes of the points of this mass-less beam as a function of the distance $x$ of the respective point from $m_1$ (longitudinal coordinate of the fuselage). In the case considered here, the vibrations of the masses $m_1$ and $m_2$ belonging to the natural frequency $\omega_1$ show a phase difference of $\pi$ whereas they oscillate in the same phase for the natural frequency $\omega_2$.

The calculations were carried out so as if they concerned an experiment; only such values appear as could be determined in an experiment. (The structural values were utilized for the calculation of amplitude changes and frequency displacements resulting from the fixation of the added masses.)

We know that we can determine the mass $m$ of a vibrator when we measure the displacement of the natural frequency by the added mass $\Delta m$.

If $\omega$ designates the original frequency and $\bar{\omega}$ the new frequency (2.3), we obtain

$$m = \Delta m \frac{\bar{\omega}^2}{\omega^2 - \bar{\omega}^2} \tag{6}$$

These relations are, in expanded form, also valid for natural vibration forms of systems with arbitrary mass and stiffness distribution if the added masses are proportional to the existing masses (adapted added masses). For sufficiently small added masses, adaptation is not required, e.g. one added mass is sufficient. A prerequisite herein is that the natural vibration form does not noticeably change by fixation of this added mass.

For a natural-vibration form assumed as non-variable, mass and stiffness can be grouped in one reference point so that nothing is changed concerning the energy regime of the system in regard to the natural-vibration form considered. With this, the system is replaced, for the respective natural-vibration form, by a simple vibrator which imparts to the system (assumed to be mass-less) the deformation prescribed by the natural-vibration form. The properties of the vibrator obviously change with the reference point (4). We call the value m obtained by equation 6 for the reference point "the mass reduced to the reference point" of the system for the natural-vibration form under consideration. By normalizing the amplitude in the reference point to 1 (m), we obtain the generalized mass $\mu$. The numerical values of m and $\mu$ are equal in this so that equation 6 may also be written in the form

$$\mu = \Delta m \frac{\bar{\omega}^2}{\omega^2 - \bar{\omega}^2} \tag{6a}$$

Whereas the value $\frac{\omega^2 - \bar{\omega}^2}{\bar{\omega}^2}$ changes proportionally to $\Delta m$ for the simple vibrator so that, first, the value obtained from equation 6 becomes independent of $\Delta m$ and, second, the function $\frac{\omega^2 - \bar{\omega}^2}{\bar{\omega}^2}(\Delta m)$ results in a straight line, this linear relation for the natural-vibration form of an arbitrary system is obtained only then when the added masses have been adapted. For non-adapted added masses, the natural-vibration form changes and the "theoretical straight line" becomes a curve whose starting tangent for $\Delta m = 0$ coincides with this

straight line. The value $m = \Delta m \frac{\varpi^2}{\omega^2 - \varpi^2}$ also does not remain constant and we obtain $m = f(\Delta m)$.

It has been shown in practice that a very restricted number of small and appropriately distributed added masses is sufficient to determine the $\mu$-values in the experiment. (Hereafter, $\mu$ will be written in place of m where natural-vibration forms are concerned.) The criterion for this is constituted by the fact that, within the range of accuracy of measurement, $\mu$ becomes independent of $\Delta m$ and/or $\frac{\omega^2 - \varpi^2}{\varpi^2} (\Delta m)$ becomes a straight line.

This now constitutes an inducement to utilize the deviations from these conditions observed with arbitrarily arranged added masses for drawing conclusions as to the internal structure of the system. Among the possible information, we are here interested in the coupling between the individual structural elements. We here proceed in the following manner:

We assume that the natural-vibration forms of the system in Fig. 2 are known from tests. By affixing added masses at two arbitrarily selected points of the system, we follow the deviations from the theoretical value $\mu$ = const. (Fig. 3-6) and, in the other representation (Fig. 7-8), from the theoretical straight line for adapted added masses. Because computation is here intended to replace a test, the theoretical values for $\mu$ as well as for the straight line are assumed as unknown and were not drawn in.

In the system represented in Fig. 2, we first affix added masses at $m_1$ and then at $m_2$ and determine the $\mu$-values for different values of the added masses. For adapted added masses, the value of $\mu$ calculated from equation 6 is independent of the value of the added mass affixed. The poorer the adaptation is, the more do the deviations from the theoretical value increase with increasing $\Delta m$. Figs. 3 and 4 show the results for the natural frequency $\omega_1$. The masses were affixed in $m_1$ (Fig. 3) and in $m_2$ (Fig. 4) and selected so that the frequency changes in both cases became about the same because these are the only base for the selection of the magnitude of the added masses in a test. In addition to the curves $\mu(\Delta m)$, the curves $\mu \cdot \beta$ were drawn in with

$$\beta = \left(\frac{A_r}{A_s}\right)^2_{\Delta m = 0} \cdot \left(\frac{A_s}{A_r}\right)^2_{(\Delta m)} , \qquad (7)$$

r = 1; s = 2 for $\mu$ in relation to $m_1$ (Figs. 3,4),
r = 2; s = 1 for $\mu$ in relation to $m_2$ (Figs. 5,6,7,8).

In this manner, we enclose the real $\mu$-values between two curves. We see from Fig. 3 that the uncertainty range so defined becomes very small for the frequency changes (less than 5%) of practical interest. However, in Fig. 4, it becomes very great and strongly dependent on the coupling.

From this we can draw the following conclusion: in a test, if the added masses are affixed successively at the structural elements to be investigated concerning their coupling and if in both cases, at the same magnitude of frequency displacements, a small range of uncertainty results, we are in the presence of firm coupling. If, in one case, a small and, and in another case, a large range of uncertainty results, we are in the presence of weak coupling; obviously, the measured $\mu$-value is determined in essence through the element which produces the smaller range of uncertainty.

Corresponding results are obtained for the natural frequency $\omega_2$ (Figs. 5 and 6).

It should be noted further that the representation in Figs. 3-6, selected because of its clear arrangement, is not the one customary in the test. Because of the scatter of the experimental results, there is no point in calculating the $\mu$-values from equation 6 as was done in Fig. 3-6. On the contrary, $\frac{\Delta\omega^2}{\omega^2}$ is entered in dependence on $\Delta m$ and the gradient of the theoretical straight line produces $1/\mu$. In Figs. 7 and 8, the results of Figs. 5 and 6 are recorded in this form utilized in the test. The theoretical straight line coincides with the starting tangent of the curve (with non-adapted added masses). The enclosing of this straight line by two curves is here especially advantageous.

## 5. LITERATURE REFERENCES


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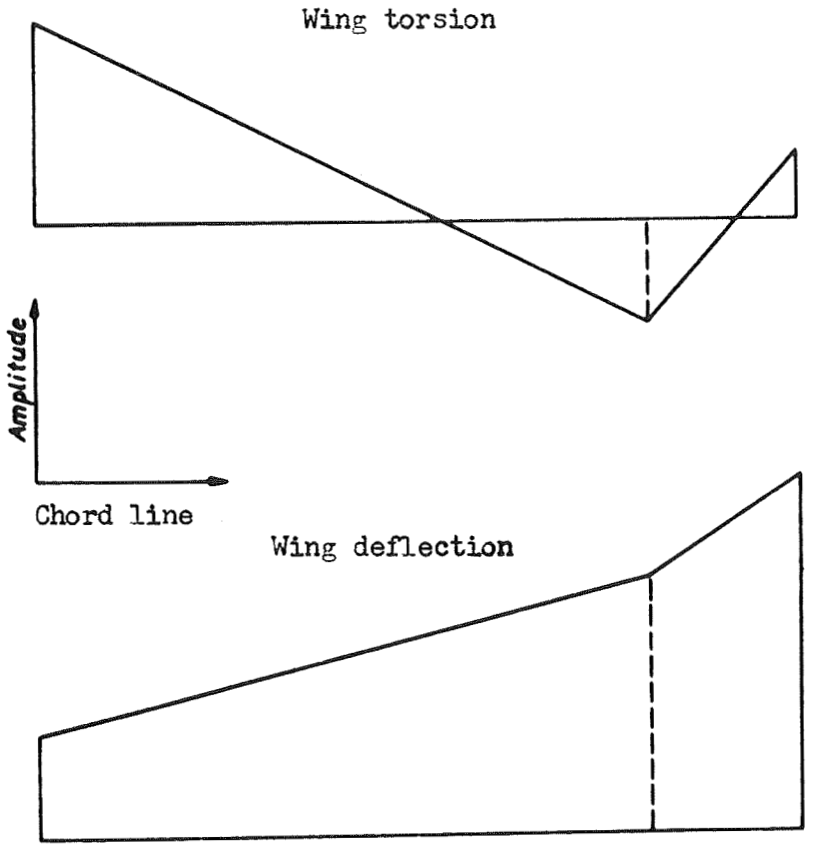



Fig. 1. Deformations for two modes of natural vibration in a reference section of a wing with control surface.

Part I of Fig. 2:

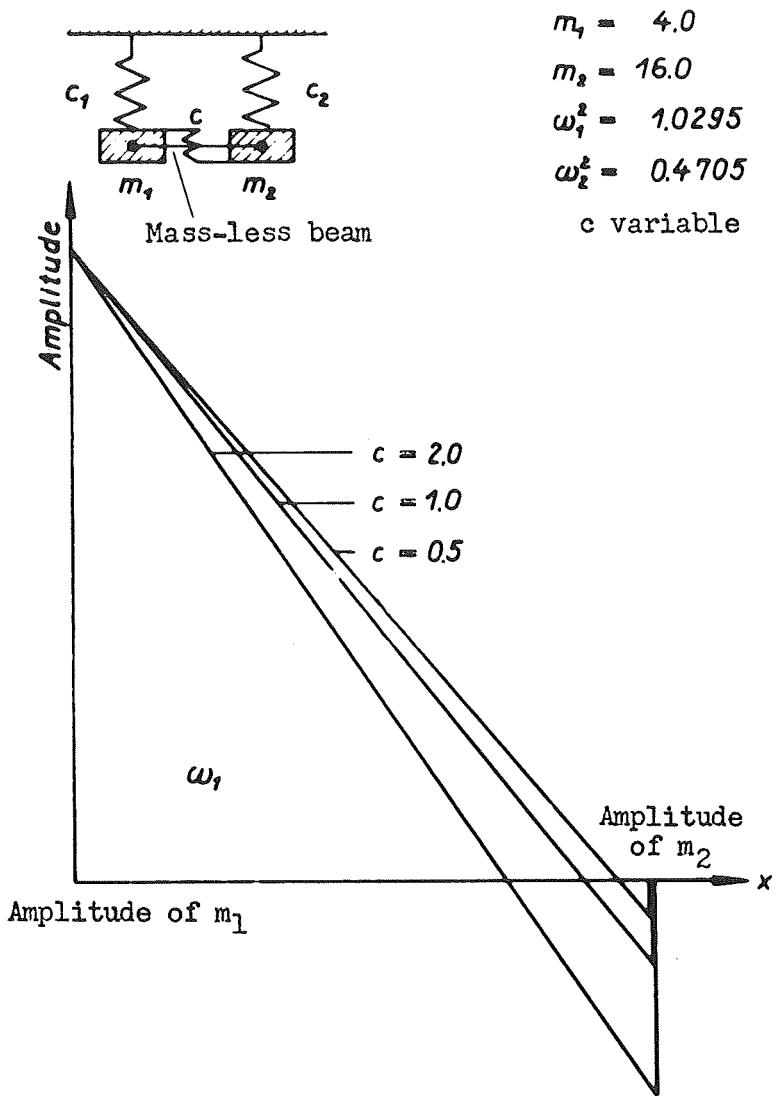
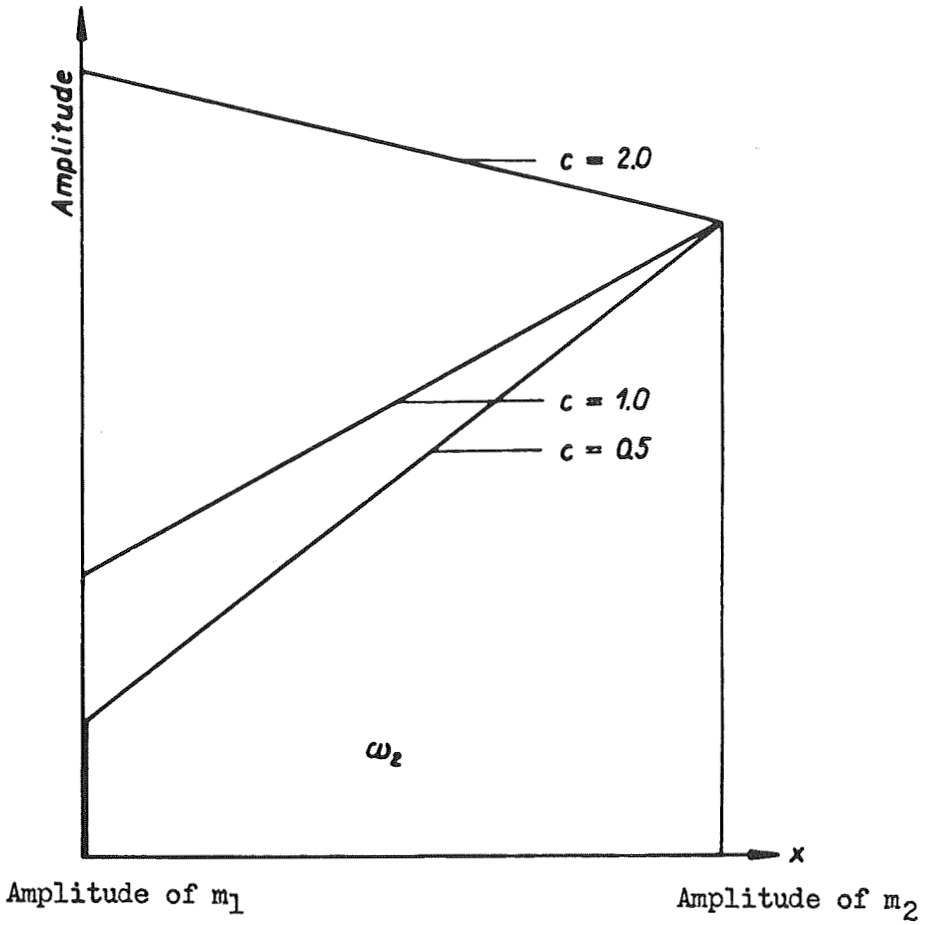



Fig. 2. Amplitudes for the natural vibration modes for different values of c.

Part II of Fig. 2:



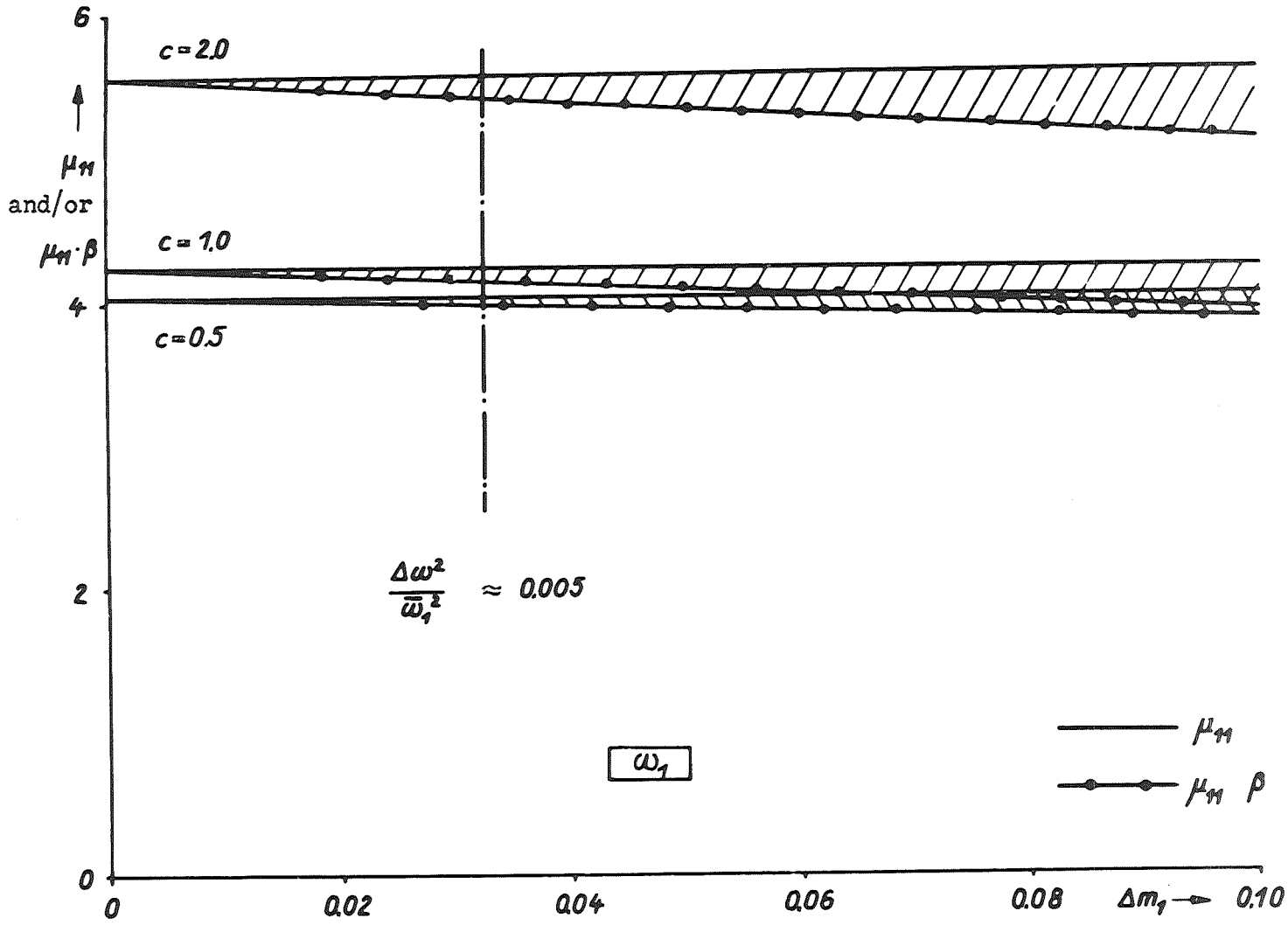



Fig. 3. Values of $\mu_{11}$ in dependence on $\Delta m_1$ for different values of c.

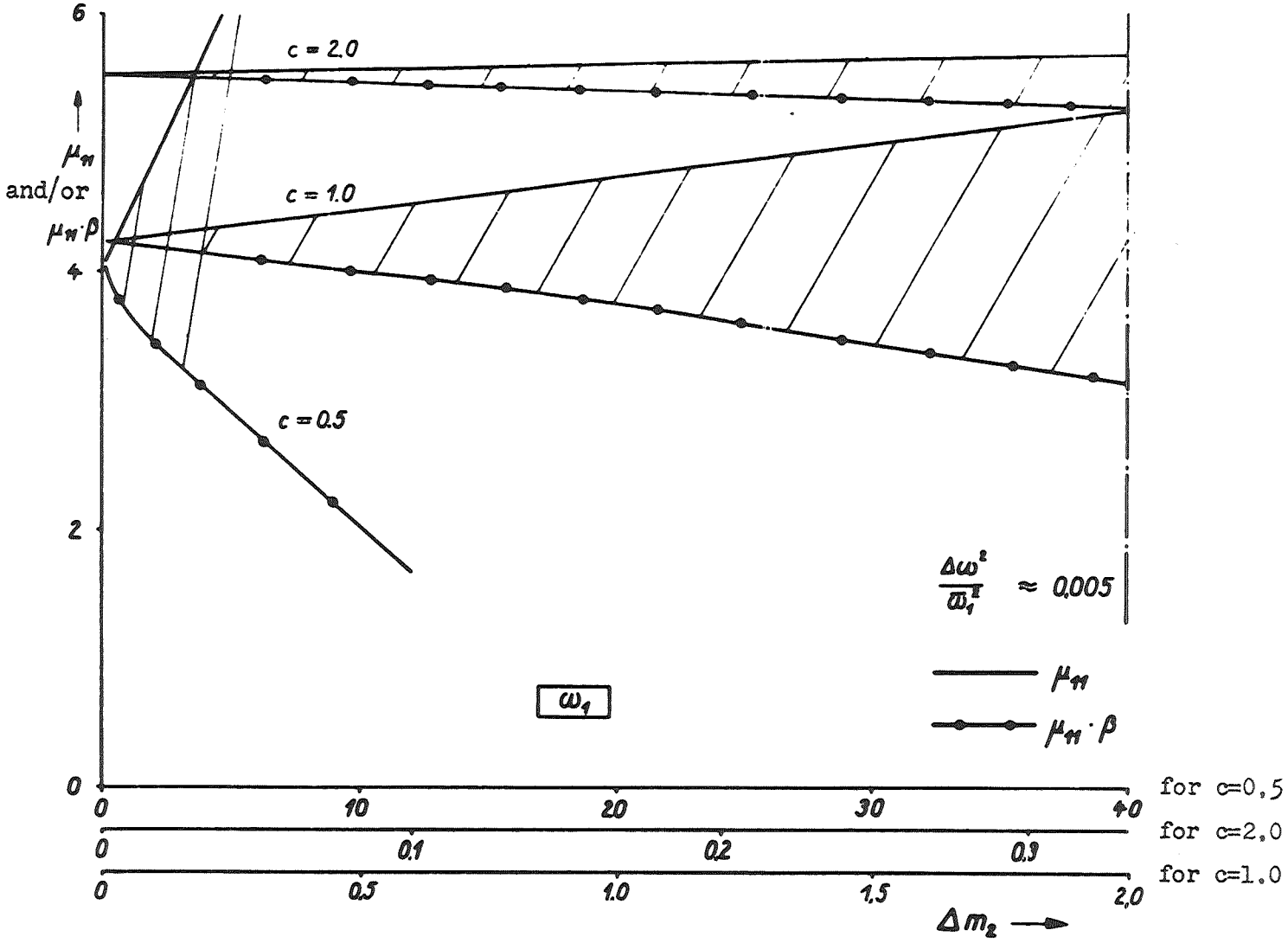



Fig. 4. Values of $\mu_{11}$ in dependence on $\Delta m_2$ for different values of c.

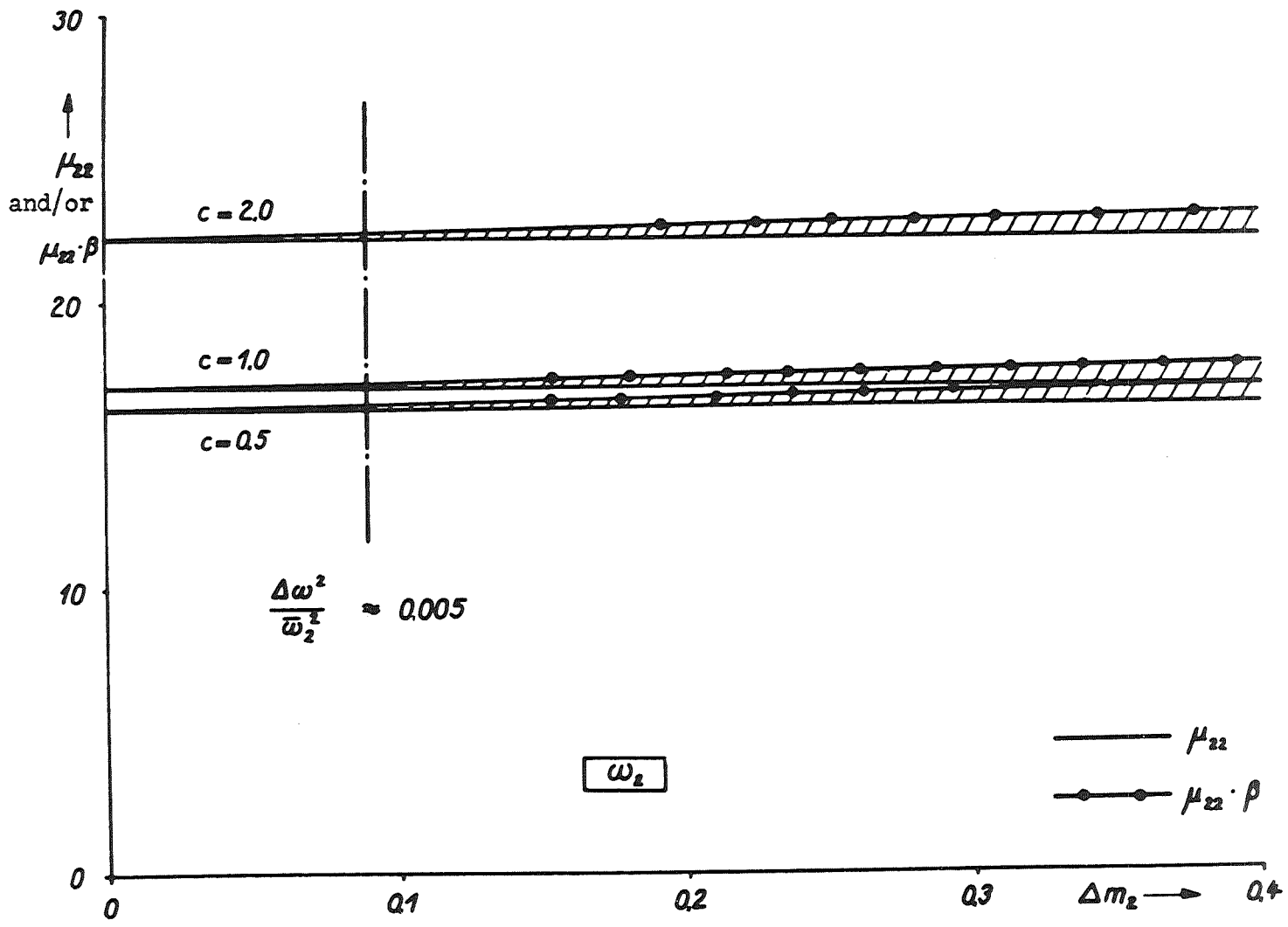



Fig. 5. Values of $\mu_{22}$ in dependence on $\Delta m_2$ for different values of c.

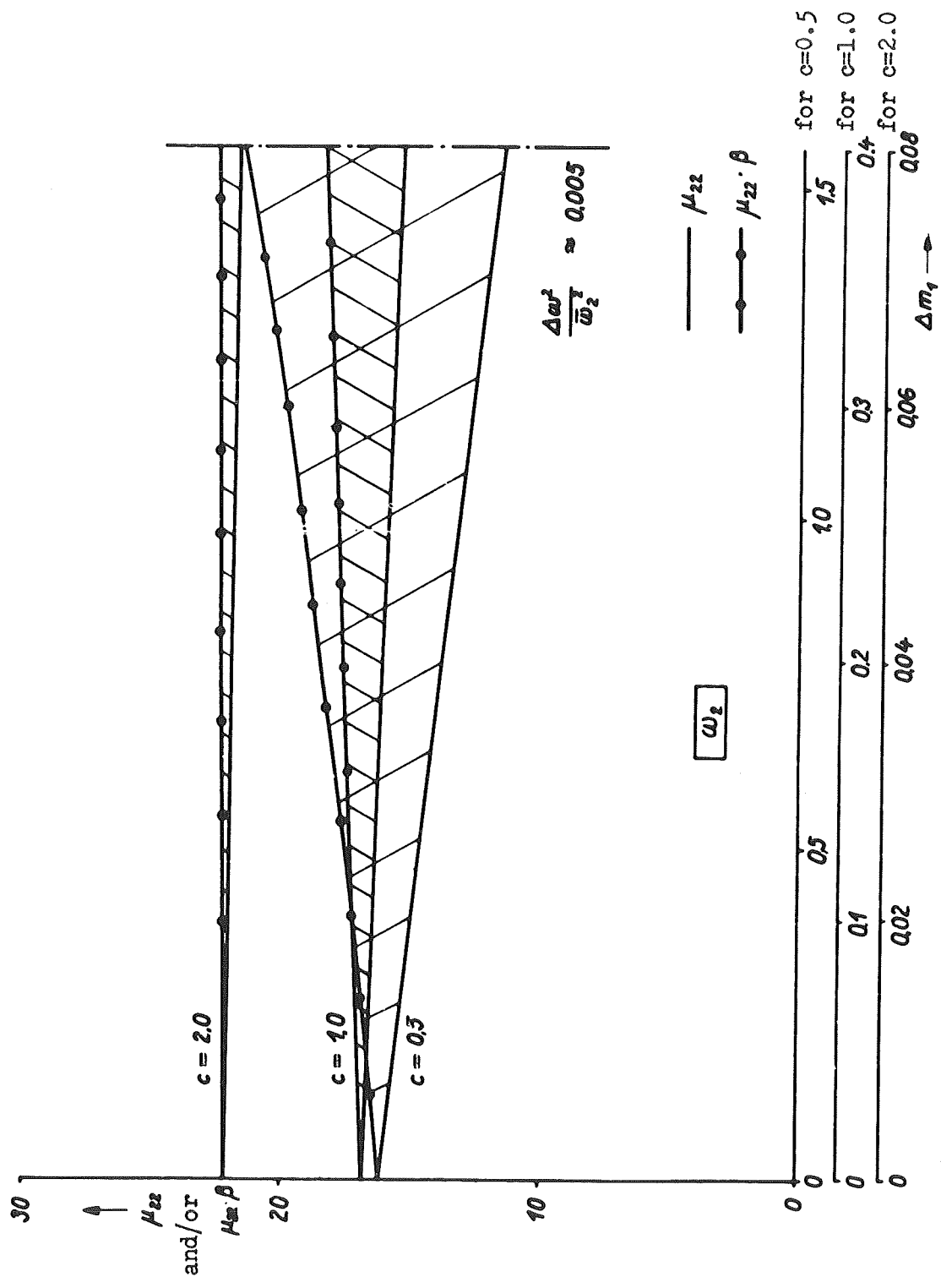



Fig. 6. Values of $\mu_{22}$ in dependence on $\Delta m_1$ for different values of c.

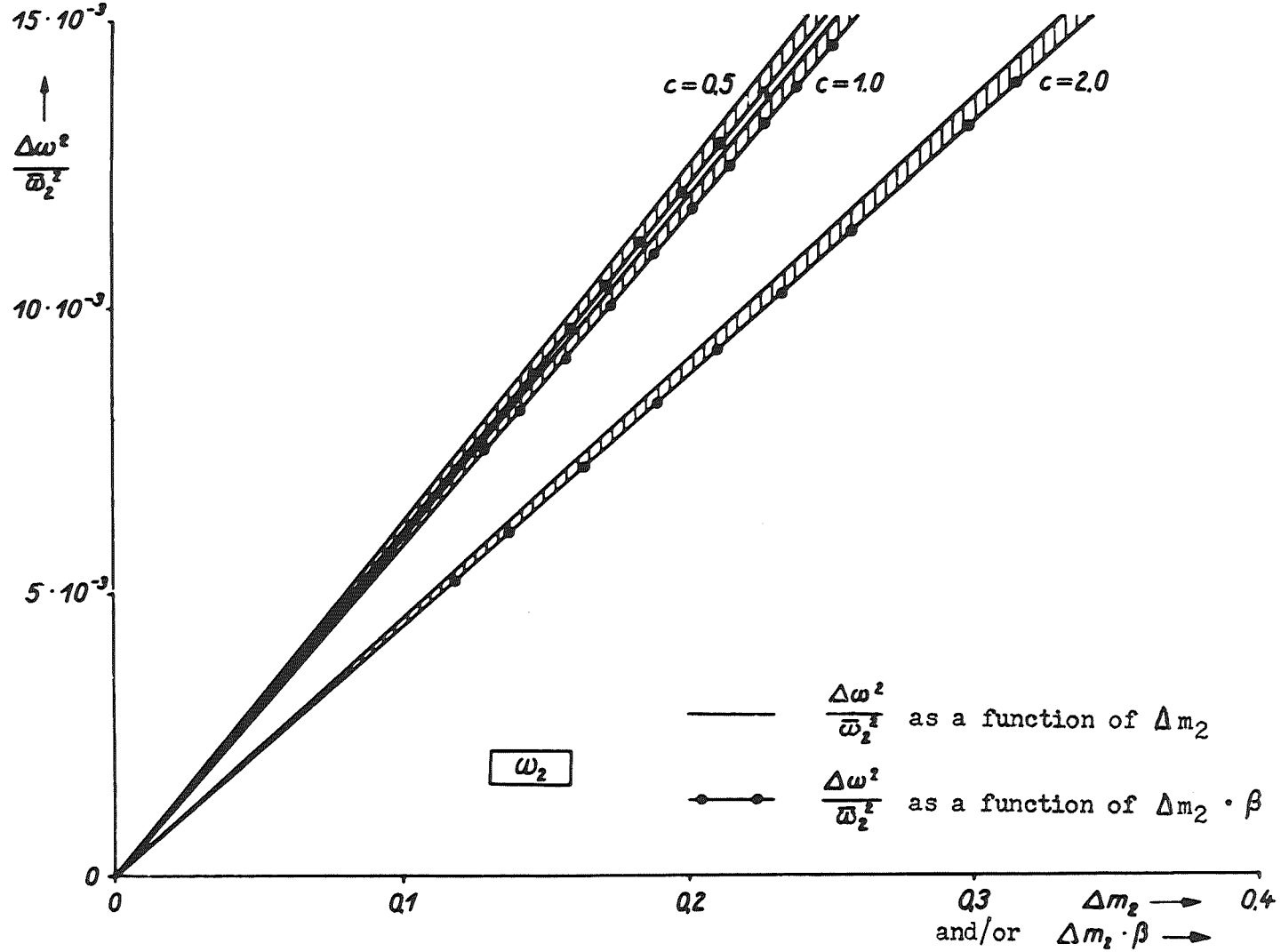



Fig. 7. Customary representation of the results in Fig. 5.

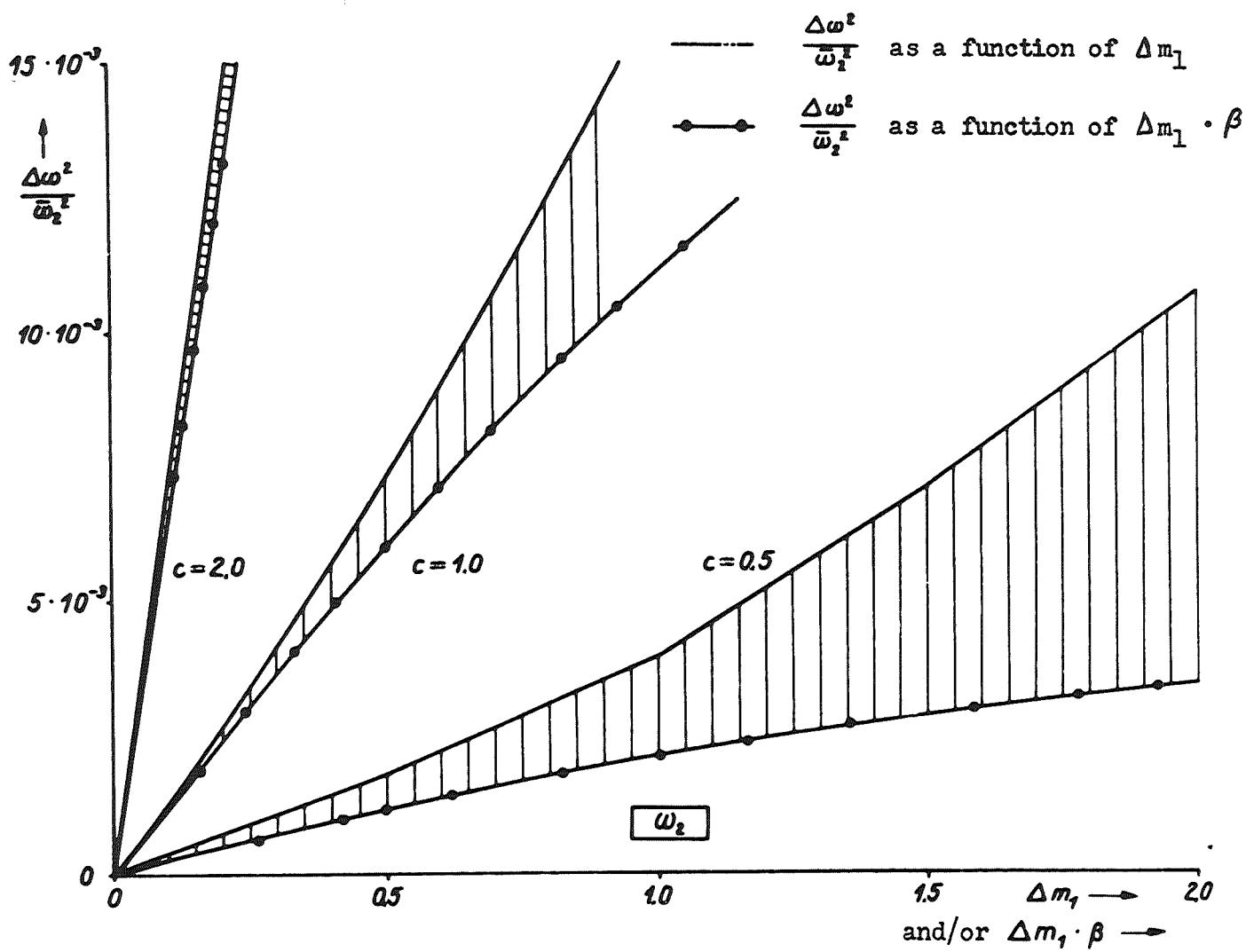



Fig. 8. Customary representation of the results in Fig. 6.

DISCUSSION

Küssner: Especially recently, we have occupied ourselves intensively with the determination of the generalized mass which is fundamental in the calculation of flutter. The investigations of Mr. de Vries now show indirectly the existence of possibilities of error which concern the resonance situation in a totally unexpected manner. It is not sufficient to comply with the phase conditions for two points of the aircraft but we must pose even stricter conditions. One of the means for this is that of introducing artificial errors into the distribution of the added masses, i.e., to affix them in the reverse sense from that corresponding to a proportional distribution. This affords us indirectly an insight into the overlapping of natural-vibration forms. It is important for us to realize that we have not yet reached our final goal but must employ much stricter criteria in order to obtain the higher natural-vibration forms. Otherwise we will measure some arbitrary natural-vibration forms, continue our calculations and then be surprised at the large deviations when their cause actually lies in the measurements.

One more point: can, shall or must we carry our calculation of flutter with a small number of natural-vibration forms? I should be more inclined to agree with the Dutch point of view of considering seven or eight forms in this but then also adding the degrees of freedom of rigid-body motion, i.e., the degrees of freedom of the complete aircraft, and obviously also the motions of the control surfaces as rigid body.

Greidanus: I have been aware for several years that Mr. de Vries occupies himself with generalized masses. I have wondered occasionally at the purpose of determining generalized masses by a new static-vibration test whereas an analysis of direct mass distribution is not really so very difficult. We have never found it necessary to carry out such measurements because we had to determine the mass distribution in the aircraft for many purposes and the former can be determined adequately, accurately without any great difficulties. We have always obtained the Lagrange equation as well as the flutter equation, without being forced to perform special tests for the determination of generalized mass.

de Vries: The answer to this is simple: in the static-vibration test, in general only the exterior of an aircraft is accessible to measurement. However, for example, jet engines, fuel tanks, etc. represent large masses which may have motion relative to fuselage and wing. We are therefore not justified, in integral formation, to multiply the masses with the amplitudes measured on the exterior of the aircraft. Since these masses are large, the error may also be very large. The same is true for the vibration forms of control surfaces and control mechanisms. It is easy to determine the inertial moment of a control surface. However, in order to indicate the generalized mass of

a vibration form, e.g., control stick against control surface, we would have to know the amplitude distribution along the control mechanism which obviously can hardly be measured.

For these reasons, appreciable errors (30% and more) can result in the calculation of generalized mass from amplitude and mass distribution. In a satisfactory result of measurement in which the results of both methods of measurement (energy method and added masses) agree, uncertainty is scarcely larger than $\pm$ 5%. Moreover, experimental determination can be carried out much faster than the respective calculation.

Küssner: You are right from one point of view, Dr. Greidanus. To you as aircraft builder, parameters are much more easily available. I endeavored, during the war, with my Flutter Team to calculate cases of flutter observed in flight. It was nearly impossible to collect the parameters. For someone on the outside, it is very difficult to procure the mass values. (Mountains of construction drawings). As an experimental institute, we are very pleased to have experimental procedures for the determination of generalized masses. We would prefer to avoid the calculation of stiffness because this requires numerous additional simplifying assumptions. Just as you do, Mr. de Vries, we also attempt to determine everything experimentally as far as possible. I should like to begin the calculation of flutter only after I have a static vibration test as good as possible. In industry, the situation is different (trained personnel for the determination of mass values, etc. from construction drawings).

Sorg: A remark on the question of the number of degrees of freedom in flutter calculation: during my activities with O.N.E.R.A., I found that quite satisfactory results can be achieved for aircraft with straight, i.e., not swept-back, wings and without heavy assemblies such as engines, wing tanks, etc. already with three degrees of freedom. I was able to observe that, in general, by the addition of more degrees of freedom such as top deflection, top torsion and the degrees of freedom of the rigid body of oscillation and torsion, the result of the flutter calculation for the wing was very little changed. The degrees of freedom always taken into account were basic wing deflection, basic wing torsion and control-surface torsion. In strongly swept-back wings or those with heavy assemblies as well as for empennages, however, conditions were different.

While re-calculating the factors of a crash due to flutter (rupture of the wing of a freight glider, equipped with engines subsequent to construction), we even calculated, with two degrees of freedom, a critical speed which, if memory serves me, agreed within 5% with the speed at the moment of crash. This concerned deflective and torsional wing flutter. The ailerons were not involved.

Ebner: Broadbent demonstrated in a report that a number of crashes due to flutter occurred because of not taking into consideration higher natural-vibration modes.

Küssner: The question of the number of degrees of freedom to be considered can probably not be decided in a general manner. In an earlier (heroic) time, we did carry out calculations with very few degrees of freedom which agreed quite well with the observed critical speeds. To me, it seems to be a safer method to introduce many degrees of freedom. Even if they have no effect on the vibration form, at least they do not introduce any additional errors into calculation. This may be a superfluous effort of computation, but it presents less of a risk.

de Vries: At O.N.E.R.A., we proceed in the following manner: we first establish considerations of energy through preliminary calculations (cf. C. Beatrix, "The Energy Method for the Detection of Vibration Modes Responsible for Flutter," Recherche Aeronautique, No. 59, 1957) by investigating, among about five degrees of freedom, in each case combinations of 2 and 2 degrees of freedom in regard to which mode imparts energy to other modes and how much energy is transmitted in the process. From these calculations, we are already able to note which, if any, of the modes enter into the conditions of flutter and which modes must be taken into account. Variations may be made in these calculations with two degrees of freedom by varying, for example, mass and by investigating whether and how the exchange of energy among the modes is influenced by this. We do not by any means want to restrict the number of natural-vibration forms in the calculation. We measure as many as 24 modes but can obviously not introduce all modes simultaneously into the calculation. These simple calculations therefore afford us a method of selection. After the preliminary calculations have been completed, the major calculation is carried out with the 5, 6 or 8 modes which are of particular interest. This final calculation then becomes controlling on the designer. It has been shown so far that not more than 5 degrees of freedom are of influence in any calculation. Normally, 3 to 4 degrees of freedom are the maximum. This would also seem to explain why we obtained good results in the past from calculation with very few degrees of freedom.

Thielemann: I would like to add one remark concerning the report of Broadbent mentioned by Prof. Ebner. Broadbent reported that he knew of two cases in which flutter occurred due to a controlling mode of oscillation which was not found during the vibration test. Theoretical determination of the natural vibration modes had not been made concurrently with the vibration test and there were utilized, in the calculation of flutter, only those modes found in the vibration test which produced sufficiently high critical speeds. After flutter occurred in the aircraft, mathematical determination of the vibration forms was made and it was noted that there must have existed two very closely contiguous natural frequencies, one of which was overlooked in the test and the natural vibration form of the latter was essentially responsible for the flutter in the two cases mentioned. In control tests carried out subsequently with great care, the existence of two

very closely contiguous natural vibration modes was actually established. In order to exclude such cases of flutter as far as possible, it would seem necessary to always carry out mathematical determination of the natural mode of the complete aircraft, in addition to the static vibration tests, in order to uncover any possible very close contiguity of frequency.